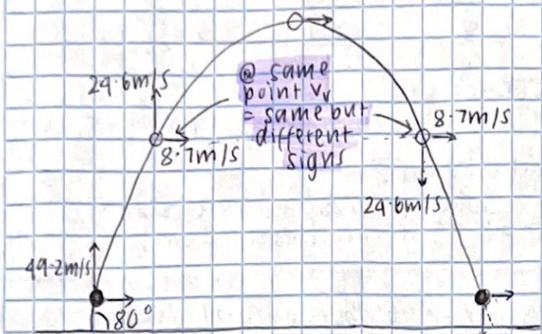
PHYSICS - projectile motion

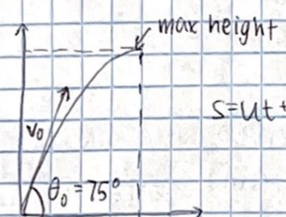
important thing to note: the horizontal motion of a launched projectile has no effect on its vertical motion (& vice versa)



horizontal velocity = same for whole flight (when no air resistance)

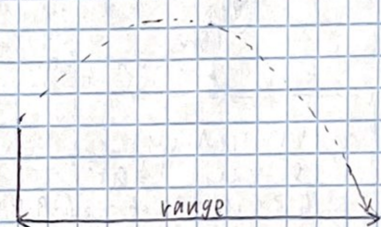
horizontal component = vector $\cos\theta$
vertical component = vector $\sin\theta$
θ = angle adjacent to horizontal component

VERTICALLY WORKING



$s = ut + \frac{1}{2}at^2$

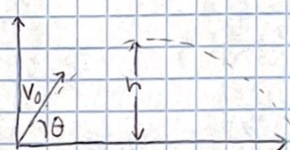
HORIZONTALLY WORKING



$s = ut$

COMBINED WORKING

2 unknowns:
- simultaneous equations
- time links them

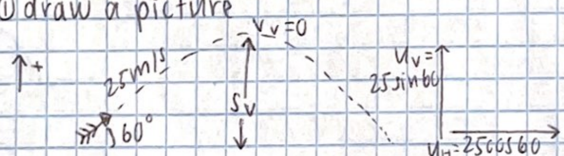


example question vertical type

arrow is fired into the air at a speed of 25m/s at an angle of 60° to the horizontal

a) how high will the arrow go? b) what is the arrow's speed after 3s?
c) where will the arrow land?

① draw a picture



a) VERTICALLY

$v_v^2 = u_v^2 + 2as_v$

$s_v = \frac{v_v^2 - u_v^2}{2a}$

$= \frac{0 - (25\sin 60)^2}{2 \times (-9.8)}$

$= 23.9\text{m}$ (above where you started)

b) $t = 3s$

$v_H = 25\cos 60$ (doesn't change)

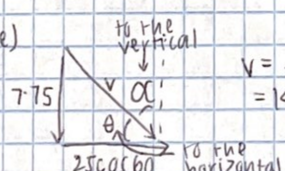
$v_v = ?$

VERTICALLY

$v_v = u_v + at$

$= 25\sin 60 + (-9.8)(3)$

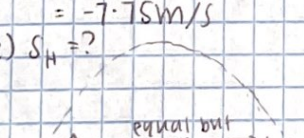
$= -7.75\text{m/s}$



$v = \sqrt{7.75^2 + (25\cos 60)}$

$= 14.7\text{m/s}$

c) $s_H = ?$



$v_H = 25\cos 60$

$s_v = 0$

HORIZONTALLY

$s = ut + \cancel{\frac{1}{2}at^2}$ a=0

$s = ut$

$= 25\cos 60 \times 4.42$

$= 55.2\text{m}$

VERTICALLY

find t first

$v_v = -25\sin 60$

$v_v = u_v + at$

$t = \frac{v_v - u_v}{a}$

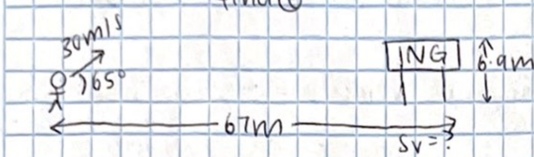
$\frac{(-25\sin 60) - (25\sin 60)}{-9.8}$

$= 4.42s$

example question horizontal type

sign is 67m away & is 6.9m above the ground. $v = 30m/s$, $\theta = 65°$, will it hit it?

find (t)



$u_H = 30\cos(65)$

$S_H = u_H t$

$t = \frac{S_H}{u_H} = \frac{67}{30\cos 65} = 5.28s$

VERTICALLY

$S_V = ?$, $u_V = 30\sin 65$, $a = -9.8$, $t = 5.28s$

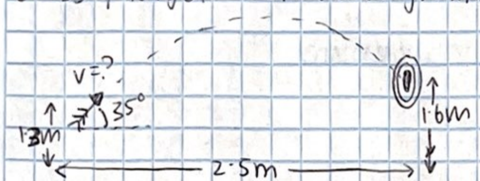
$S = ut + \frac{1}{2}at^2$

$= 30\sin 65(5.28) + \frac{1}{2}(-9.8)(5.28)^2$

$= 6.95m$ (misses by 5cm)

example question combined type

$\theta = 35°$, target = 2.5m above ground, thrown from 1.6 above ground, find v



$a = -9.8 m/s^2$

$u = ?$ — $u_H = u\cos 35$, $u_V = u\sin 35$

$S_H = 2.5m$

$S_V = 0.3m$

HORIZONTALLY

$S_H = u_H t$

$2.5 = u\cos 35 \times t$

$t = \frac{2.5}{u\cos(35)}$

VERTICALLY

$S_V = u_V t + \frac{1}{2}at^2$

$0.3 = u\sin 35 \times t + \frac{1}{2}(-9.8)t^2$

COMBINED EQUATION

$0.3 = \cancel{u}\sin 35 \times \frac{2.5}{\cancel{u}\cos 35} - 4.9\left[\frac{2.5}{u\cos 35}\right]^2$

$\frac{\sin}{\cos} = \tan$ so $\frac{\sin 35}{\cos 35} = \tan 35$

$0.3 = 2.5\tan 35 - 4.9 \times \frac{2.5^2}{u^2\cos^2 35}$

$\frac{\overbrace{4.9 \times 2.5^2}^{30.625}}{u^2\cos^2 35} = \overbrace{2.5\tan 35 - 0.3}^{1.451}$

$\frac{30.625}{u^2\cos^2 35} = 1.451$ $\quad \frac{30.625}{1.451} = u^2\cos^2 35$

$21.106 = u^2\cos^2 35$ $\quad 21.106 = u^2 \times 0.671$

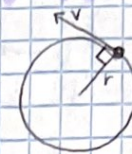
$\sqrt{\frac{21.106}{0.671}} = u$ $\quad u = 5.6m/s$

PHYSICS - HORIZONTAL circular MOTION

3 kinds of HCM questions:
- ball on a string
- banked track
- leaning into a corner

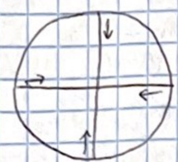
average speed:

$$v = \frac{\text{distance}}{\text{time period}} = \frac{2\pi r}{T}$$



v = circular speed (m/s)
r = radius (m)
T = period (time of 1 rotation) (s)

centripetal force:



(i.e acceleration towards the centre)

$$a_c = \frac{v^2}{r}$$

$$F_c = ma_c = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2}$$

DERIVATION

$F_c = ma_c$

$= m \times \frac{v^2}{r}$

$v = \frac{s}{t}$

$v = \frac{2\pi r}{T}$

$= \frac{4\pi^2 r^2}{T^2}$

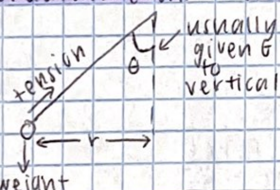
$F = m \times \left(\frac{\frac{4\pi^2 r^2}{T^2}}{r}\right)$

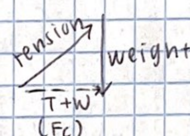
$F = \frac{4m\pi^2 r}{T^2}$

F_c = not a real force

system	source of F_c
ball on a string	tension force of string
moon in orbit	gravitational force
electron in orbit	electrostatic force
circling ice skater	normal reaction force of ice
racing car turning on banked track	normal reaction force of track

pendulum (ball on string)

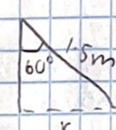




example

$m = 57g$ $\quad \ell = 1.5m$

$\theta = 60°$ to vertical

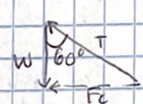


a) find r

$r = 1.5\sin 60$

$= 1.3m$

b) determine net force



$\tan(60) = \frac{F_c}{W}$

$F_c = W\tan(60)$

$= mg\tan 60$

$= 0.968N$ towards centre

c) find size of T

$\frac{W}{T} = \cos(60)$

$T = \frac{mg}{\cos 60}$ $\quad T = 1.12N$

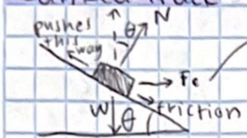
d) $v = ?$

$F_c = \frac{mv^2}{r}$ $\quad F_c r = mv^2$

$\frac{F_c r}{m} = v$ $\quad v = \sqrt{\frac{F_c r}{m}}$

$v = \sqrt{\frac{0.968 \times 1.3}{57 \times 10^{-3}}}$ $\quad v = 4.7m/s$

banked track



due to the component of the reaction force of the track on the car

to calculate banking angle required for a particular speed:

not on the formula sheet → $v = \sqrt{rg\tan\theta}$

DERIVATION

$\tan\theta = \frac{F_c}{w}$

$F_c = w\tan\theta$

$F_c = mg\tan\theta$

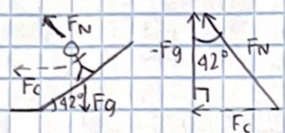
$\frac{\cancel{m}v^2}{r} = \cancel{m}g\tan\theta$

$\frac{v^2}{r} = g\tan\theta$

$v^2 = rg\tan\theta$

example

$r = 50m$, $\theta = 42°$, $m = 75kg$



a) find F_c

$\tan\theta = \frac{F_c}{w}$

$F_c = w\tan\theta$

$= 75(9.8)\tan(42)$

$= 661.79N$

$= 662N$

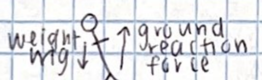
b) find design speed

$v = \sqrt{rg\tan\theta}$

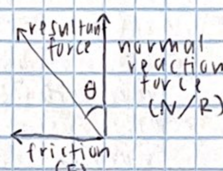
$= \sqrt{(50)(9.8)\tan(42)}$

$= 21m/s$

leaning into a bend



$F_c = F_F$



b) Find θ (banking angle) for $v = 12m/s$

$\tan\theta = \frac{F_c}{w} = \frac{\cancel{m}v^2}{r\cancel{m}g} = \frac{v^2}{rg}$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$\theta = \tan^{-1}\left(\frac{12^2}{50 \times 9.8}\right) = 16.4°$

example

$m = 110kg$, $r = 50m$

a) ~~find~~ if $F_F = 600N$, find θ

$\tan\theta = \frac{F_c}{w}$ $\theta = \tan^{-1}\left(\frac{600}{110 \times 9.8}\right)$

$\theta = 29°$

example 2

$m = 160$ tonnes, $r = 350m$, $v = 18m/s$

a) find a as it rounds the bend

$a = \frac{v^2}{r} = \frac{18^2}{350} = 0.926m/s$

or for b

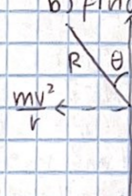
$N^2 = F_c^2 + W^2$

$N = \sqrt{\frac{mv^2}{r} + mg}$

$= \sqrt{\left(\frac{160\,000 \times 18^2}{350}\right) + (16\,000 \times 9.8)^2}$

$= 1.57 \times 10^6$

b) find reaction force



$\tan\theta = \frac{v^2}{rg}$

$= \frac{18^2}{350 \times 9.8}$

$= 5.4°$

$r\cos\theta = mg$

$R = \frac{mg}{\cos\theta}$

$= \frac{160 \times 10^3 \times 9.8}{\cos 5.4}$

$= 1.57 \times 10^6 N$

WAVE PARTICLE DUALITY + THE QUANTUM THEORY

'particle'

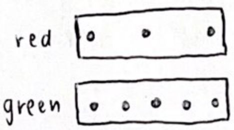
PHENOMENON	WAVE THEORY	CORPUSCULAR THEORY
reflected	✓	✓
refraction	✓	✓
interference	✓	
diffraction	✓	
polarisation	✓	
photoelectric		✓
dispersion	✓	

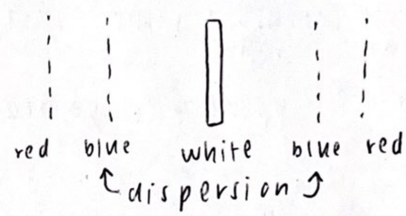
REFRACTION
- refraction always slows the wave
- air → glass = $\lambda\downarrow$
- freq. = constant

red = 690-700nm
blue = 450nm

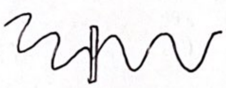
$\downarrow\lambda$ = slower, refract more
$\uparrow\lambda$ = faster, refract less

DOUBLE SLIT EXPERIMENT
- diffraction through the 2 slits
- creates interference pattern
- creates bright + dark spots
  ↳ when waves are in phase





POLARISATION
- only waves in a certain direction are allowed through
- all others are absorbed

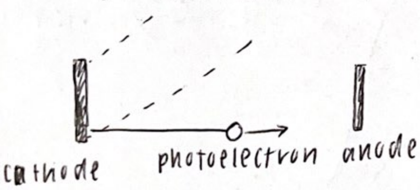


only vertical component is allowed through

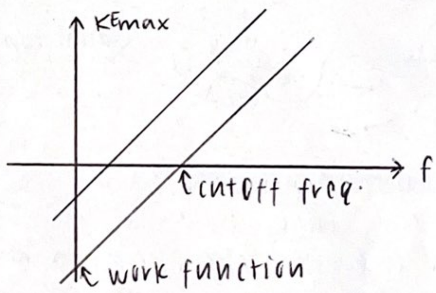
light = transverse wave

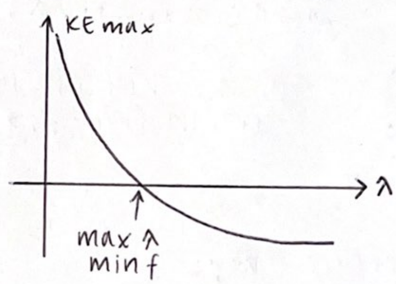
PHOTOELECTRIC EFFECT
- electrons from metal surface
- only eject when above cutoff freq.

$\uparrow I$ = $\uparrow$ photons when above cutoff f.
intensity ↳ current



instantaneously eject (should have a delay but doesn't)





W (work function) = energy to produce photoelectron

e.g. 4.2eV to release an e^-

$\boxed{W = hf_0}$

↳ threshold freq.

$KE = hf - W$

input from light - work func.

$E = \frac{hc}{\lambda}$

$v = \lambda f$ $E = hf$

if $E > W$ then photoelectron is released

× charge of e^-

eV ⇄ J

÷ charge of e^-

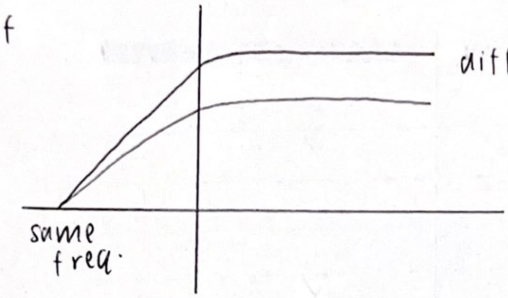
$\boxed{h = 4.14 \times 10^{-15}\ eV}$

$KE_{max} = h\frac{c}{\lambda} - W$

stopping voltage = max KE (eV)

no current @ stopping voltage

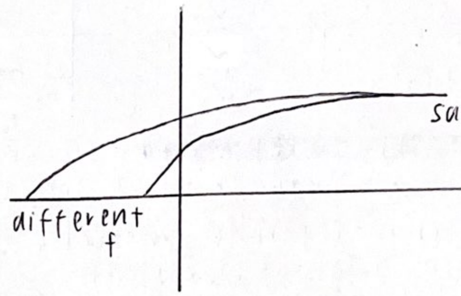
$W = hf_0$
↑ threshold f



$W = Vq$
↑ stopping V

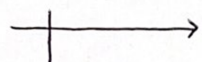
↑ work (energy) only tells fastest moving e^-'s energy

stopping V is constant for a particular f on a particular metal



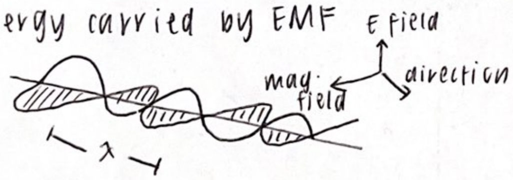
- same intensity = same max. current
- ↑f = ↑stopping V (more neg.)
- ↑f = ↑KE

$W = Vq$
$\uparrow W = \uparrow V$



v=c in a vacuum

energy carried by EMF



$W = J/s$

$P = \frac{E}{t}$

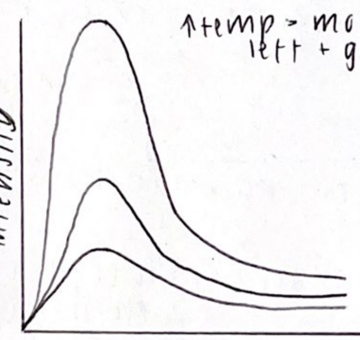
① $E = h\frac{c}{\lambda}$

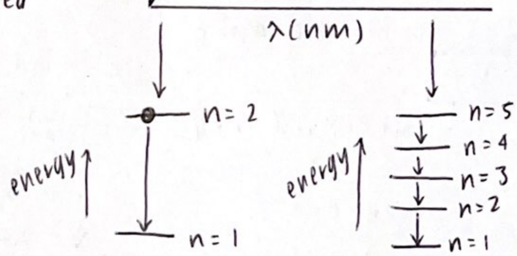
② $\frac{J/s}{\text{energy}}$

③ per min = ×60

BLACK BODY RADIATION

- emit radiation of all λ
- ideal surface that absorbs all λ of EMR
- ↑ temp = ↑ amount visible (more E in shorter λ) ↑KE ∴ more intense





$\boxed{E = nhf}$
↑ no. of photons

atoms only oscillate w/ certain freq. ∴ quantised

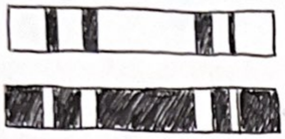
lowest surface temp.
red giant
white dwarf
blue super-giant
highest surface temp.

'packets' of particles = photons or quanta

@ threshold λ, only lower λ's work

max λ = min E

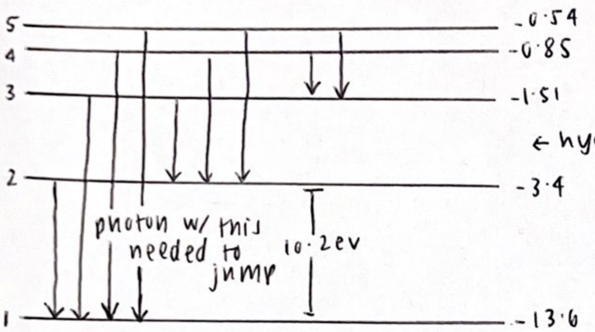
EMISSION + ABSORPTION SPECTRA



absorption

emission

EMR freq. absorbed + emitted match energy levels allowed in the atom



visible = level 2

$E = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{\Delta E}$

$= \frac{hc}{E_1 - E_2}$

BAND + CONTINUOUS SPECTRA

molecules, polyatomic ions = line/band

hot solids + liquids = continuous

when intensity is same, E is same

black body = theoretical
↳ peak → shorter λ
↳ I ↑

uv catastrophe
- light being emitted in specific amounts

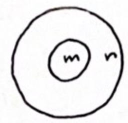
DE BROGLIE WAVES

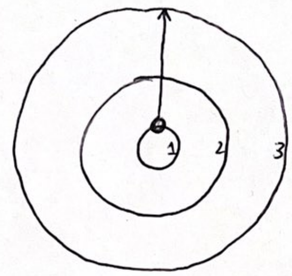
only work when they can maintain a standing wave

BOHR MODEL

- e^- emits when moving back down energy levels

$E_{photon} = hf = E_m - E_n$





e^- can only absorb from $n = 1$

only photons w/ exact energies are absorbed by the e^-

energy above ionisation → KE of e^- as it leaves

BAND SPECTRA

- caused by overlapping energy levels
- more possible E transitions

PROBLEM'S W/ BOHR'S MODEL

- only works for one e^- atoms (H or He^+)
- cannot predict higher energy level orbits of multi-e^- atoms
- cannot explain discovery of the continuous spectrum emitted by solids

DE BROGLIE λ

- FOR A PARTICLE
- IN SI UNITS

will behave as if it had a particular λ value

$\lambda = \frac{h}{p} = \frac{h}{mv}$

p ← momentum

$2\pi r = \frac{nh}{mv}$

$2\pi r$ ← circumference

SPECIAL RELATIVITY

RELATIVE MOTION

the principle of relativity:
within a frame of reference, there is no measurement you can make that will tell you whether you are stationary or moving with a constant velocity

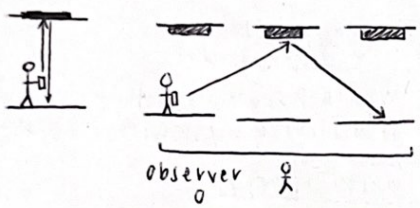
objects are always in motion relative to some other frame of reference

THEORIES OF RELATIVITY

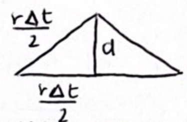
Einstein proposed 2 theories of relativity:

1. the laws of physics are the same in all reference frames
2. light always travels through a vacuum at a fixed + constant speed
   c is constant + independent of the motion of the emitting body

THE EFFECT OF SPEED ON TIME/LENGTH



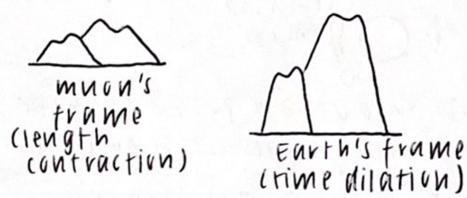
relative to O, light travels a longer distance



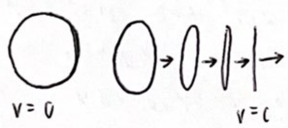
∴ time must slow down ($v = \frac{d}{t}$) - time dilation

> clocks moving relative to an observer are measured by that observer to run more slowly (as compared to clocks at rest)

e.g. muon decay



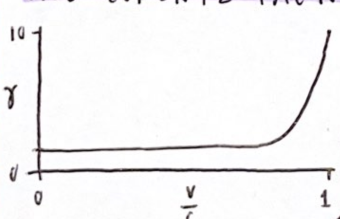
- length of an object is measured to be shorter when it is moving relative to the observer than when it is at rest
  ↳ only occurs along the direction of motion



t_0 - the time measured by the observer in the same frame of reference as the clock

l_0 - the length of the object (or distance between 2 points) as determined by the observers at rest with respect to the distance

THE LORENTZ FACTOR



as $\frac{v}{c} \to 1$
$\gamma \to \infty$
and length → 0
and time appears to stand still

$$l = \frac{l_0}{\gamma} \qquad \gamma = \frac{1}{\sqrt{(1 - \frac{v^2}{c^2})}} \qquad t = \gamma t_0$$

THE EFFECT OF SPEED ON MOMENTUM

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mv$$

at low speeds, reduces to $p = mv$

THE EFFECT OF SPEED ON ENERGY

as momentum (p) ↑, KE ↑

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(mv) \times v = \frac{1}{2}pv$$

$$E_{total} = E_0 + KE = mc^2 + (\gamma - 1)mc^2 = mc^2 + \gamma mc^2 - mc^2 = \gamma mc^2$$

RELATIVISTIC ENERGY

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

RELATIVISTIC ADDITION OF VELOCITIES

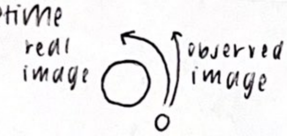
$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}} \qquad u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

u, u' = same object different reference frames

v, u = different objects, same reference frame

THE EFFECT OF GRAVITY ON LIGHT

- photons are bent by distorted spacetime



- multiple images are produced when a massive object in the foreground distorts spacetime ← gravitational lensing

THE EFFECT OF GRAVITY ON TIME

- to an external observer, time appears to run slower in stronger gravitational fields
- slower clocks on Earth vs satellites

SPECIAL THEORY OF RELATIVITY

- in cases with no gravitational fields

GENERAL THEORY OF RELATIVITY

- theory of gravitation
- unifies special relativity & universal law of universal gravitation
  - ↳ space & time → spacetime
  - ↳ spacetime's 4 dimensions:
    - length, width, height & time
  - ↳ spacetime is distorted by massive objects

$\boxed{\text{according to general theory}}$

- ↓ gravity isn't a force
- ↘ objects are attracted towards Earth because of the distortion of spacetime around it

NEWTON vs EINSTEIN

- relativistic changes in mass, length & time are insignificant

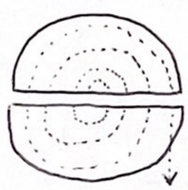
GRAVITY

- in weak gravitational fields, general relativity & Newton's laws are consistent
- in strong gravitational fields, Newton's laws cannot explain the phenomena

THE STANDARD MODEL

CYCLOTRON

- makes use of magnetic F on a moving charge to bend them into a semicircular path between accelerators
- magnetic F provides centripetal F



electric field reverses just at the time the e⁻'s finish their ½ circle

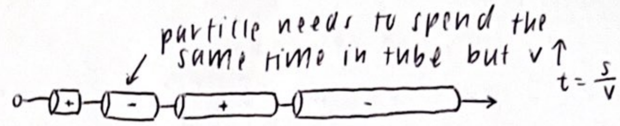
charge moves in an expanding helix:

$$F = ma \qquad qvB = \frac{mv^2}{r}$$

$$\therefore v = \frac{qBr}{m}$$

LINEAR PARTICLE ACCELERATORS (LINAC)

- e^-'s or ions are accelerated by an electric field along a straight-line path
- voltage alternates so when the e^- reaches a gap, the tube in front attracts it
- produces γ rays

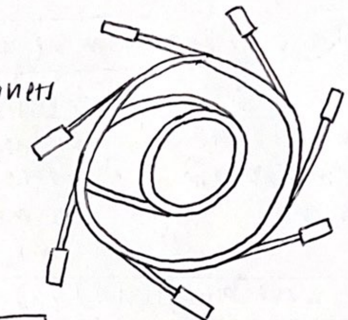


<u>voltage applied to accelerate to 99.9% the speed of light</u>

$\frac{1}{2}mv^2 = qV$ $\therefore V = \frac{mv^2}{2q} = 2.56 \times 10^5$ V

SYNCHOTRON

- e^-'s travelling at close to c under vacuum have their paths changed by powerful electromagnets
- changing paths → synchrotron light



why is synchrotron light so useful?

- **brilliant** — ↑ intensity allows researchers to collect more detailed/accurate info in a shorter time
- **polarised** — improves sensitivity
- **wide spectrum** — covers a wide range of λ's simultaneously
- **tuneable** — a single λ or range of λ's can be selected
- **collimated** — precisely focused so extremely small areas of a sample can be investigated
- **pulsed** — emitted in short flashes over a short period

PARTICLE COLLIDERS (CERN & LHC)

- allowing ↑ energy particles to strike a stationary target
- 2 particles collide head on
- guided around the accelerator ring by a strong magnetic field

<u>why build them?</u>

- same mass & magnitude
- momentum is the same ∴ cancels at collision
  ↳ ∴ KE = 0
- the energy of before the collision is transformed into the mass of the new particles formed

EVIDENCE FOR THE BIG BANG

① the expansion of the universe

<u>the doppler effect</u>

- light moving away = longer λ (red shifted)
- light moving towards = shorter λ (blue shifted)

<u>the spectra of galaxies</u>

- recorded spectra of faint galaxies

$v = \frac{\Delta\lambda}{\lambda} \times c$

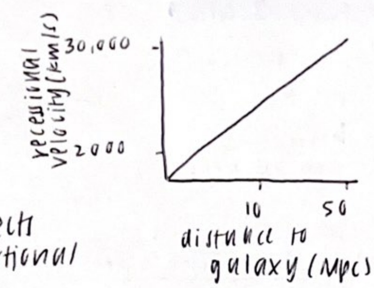
<u>velocities of galaxies</u>

- discovered that most galaxies are moving away
- used spectral shifts

hubble's law

-plotted distance vs velocity

Hubble's law: $v = H_0 D$ — distance to a galaxy; H_0: hubble's constant



the rate at which astronomical objects move apart from each other is proportional to their distance from each other

- universe is expanding

② the abundance of H and He in the universe

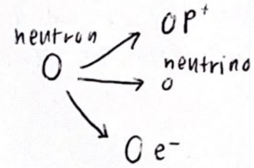
nucleosynthesis

- protons & neutrons fused to form deuterium & helium in the first few mins

neutrons

as the universe cooled, neutrons:

- decayed → p^+ and e^-
- combined w/ p^+ → deuterium
- one He for each 12 H nuclei



③ cosmic microwave background radiation

- universe became transparent about 370,000 years after the Big Bang
- photons from this time still fill the universe, but λ is ↑ to microwave EMR as the universe expanded

the cosmological principle

- CMBR shows that matter in the universe has uniform density (homogeneous) and is the same in all directions (isotropic)

CMBR

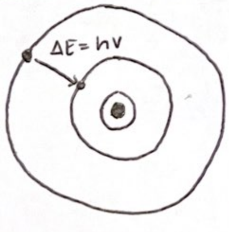
- remnant photons from the early universe
- red shifted to microwave spectra

THE STORY OF QUARKS

collisions in the LHC release energy → mass (according to $E=mc^2$)

- atom is mostly empty space

Bohr-Rutherford Atom



A Neutron

- Chadwick discovered the neutron

QUARKS

proton

2 ups & a down

neutron

2 downs & an up

quark	up	charm	top
charge	+2/3e	+2/3e	+2/3e
mass	2.5MeV/c²	1270MeV/c²	171000MeV/c²
quark	down	strange	bottom
charge	-1/3e	-1/3e	-1/3e
mass	5MeV/c²	105MeV/c²	4200MeV/c²

HADRONS

- quarks only exist in groups
  - ↳ held together by strong force mediated by gluons
- particles made of quarks are called hadrons
  - ↳ hadrons have no net integer electric charge
  - ↳ & no net colour change

BARYONS + MESONS

2 classes of hadrons:

Baryons

- 3 quarks (qqq) or 3 antiquarks ($\bar{q}\bar{q}\bar{q}$)

Mesons

- 1 quark & 1 antiquark ($q\bar{q}$)

THE STANDARD MODEL

quarks

- charm, strange, top & bottom are heavy quarks, rapidly decay to up and down quarks

leptons

- include e^- and neutrinos
- do not experience strong force ∴ can exist by themselves

force carriers

- bosons are force carriers
- photons → electromagnetic
- W & Z → weak
- gluons → strong

THE HIGGS BOSON

- responsible for giving other particles their mass
- produced when huge amounts of energy are released

CHARGE

electrostatic force of repulsion between 2 protons 1.6×10^{-15}m apart in the nucleus of an atom is 90N

STRENGTH OF FORCES

strongest

strong

electromagnetic

weak

gravitational

weakest

$(1 : 10^{-2} : 10^{-6} : 10^{-38})$

	gravity	weak	electromag.	strong
mediated by	graviton	W^+ W^- Z^0	photon	gluon
acts on	all	quarks & leptons	quarks & charged leptons and W^- W^+	quarks & gluons